Calculus AB

2-6

Related Rates

13) Find the rate of change of the distance between the origin and a moving point on the graph of $y = x^2 + 1$ if $dx/dt = 2$ centimeters per second. (pg 149)

$\frac{dd}{dt}$ — derivative with respect to time

$$d = \sqrt{x^2 + f^2(x)}$$

$$d = \sqrt{x^2 + (x^2+1)^2}$$

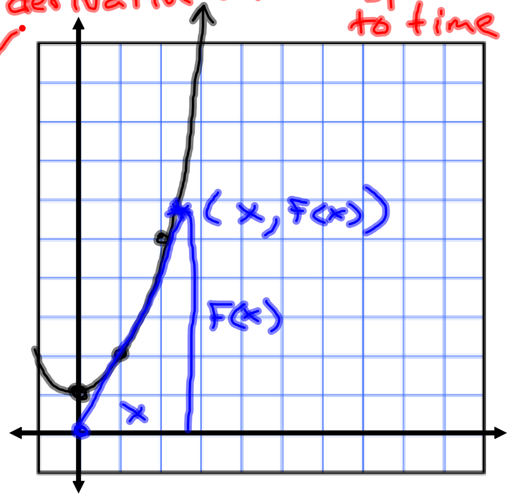
$$d = \sqrt{x^2 + x^4 + 2x^2 + 1}$$

$$d = \sqrt{x^4 + 3x^2 + 1}^{\frac{1}{2}}$$

$$\frac{dd}{dt} = \frac{1\left(4x^3\frac{dx}{dt} + 6x\frac{dx}{dt}\right)}{2\sqrt{x^4+3x^2+1}}$$

$$\frac{dd}{dt} = \frac{4x^3+6x}{2\sqrt{x^4+3x^2+1}} \cdot \frac{dx}{dt} \quad (2)$$

$$= \frac{4x^3+6x}{\sqrt{x^4+3x^2+1}}$$



23) At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 ft high?

$\frac{dV}{dt} = 10\ \frac{ft^3}{min}$

$\frac{dx}{dt}$

need x's

$V = \frac{1}{3}\pi r^2 h$

Get the equation in terms of x and V

$V = \frac{\pi}{3}\left(\frac{3}{2}x\right)^2 x$

$V = \frac{9\pi}{12}x^3 = \frac{3\pi}{4}x^3$

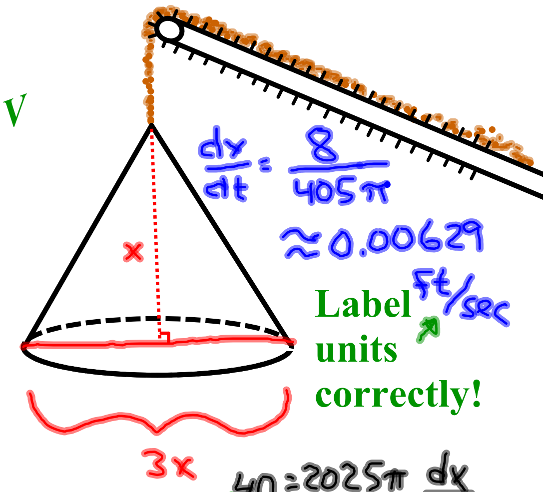
differentiating gives us $\frac{dV}{dt}$ & $\frac{dx}{dt}$

$\frac{dV}{dt} = \frac{9\pi}{4}x^2\frac{dx}{dt}$

$10 = \frac{9\pi}{4}x^2\frac{dx}{dt} \Rightarrow 10 = \frac{9\pi}{4}(15)^2\frac{dx}{dt}$

$40 = 2025\pi\frac{dx}{dt}$

$\frac{8}{405\pi} = \frac{dx}{dt}$



$\frac{dx}{dt} = \frac{8}{405\pi} \approx 0.00629$ ft/sec

Label units correctly!

Process for Related Rates Problems.

1) When reading the problem, determine variables and changing rates.

2) Once all the variables and rates of change are determined, try to develop an equation that relates all the variables and whose derivative will contain the rates of change needed.

3) A picture is often very helpful.

4) Solve.

Assignment:

Pg. 149

14-24 all